Section 6.3

General Slicing Method

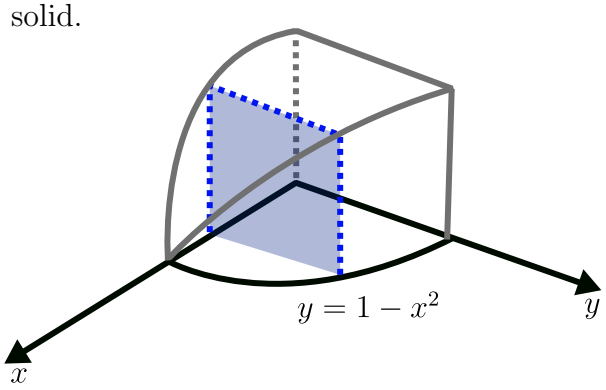
Given a prism with base B and height h, the volume of the prism can be calculated by:

Recalling our work in Section 6.1, this relation immediately suggests for a solid extending in the x direction from $x = a$ to $x = b$ with cross-sectional area $A(x)$, the volume of the solid is given by

$$V = \int_a^b A(x)dx.$$

Notice in the case of the prism, $A(x)$ is constant and we recover $V = Ah$.

Example 6.3.1. Let R be the region in the first quadrant bounded by the coordinate axis and the curve $y = 1-x^2$. A solid has base R, and cross sections through the solid perpendicular to the base and parallel to the y-axis are squares. Find the volume of the solid.



Example 6.3.2. Find the volume of a sphere of radius r using the slicing method.

Cross sections which are circular or in the shape of a washer are useful in two regards: the cross-sectional area is easy to compute and the solids to which we may apply them become more exotic.
First, a washer with inner radius r_0 and outer radius r_1 has a total area of

$$A = \pi(r_1^2 - r_0^2).$$

We may think of a circular disc as a washer without an inner hole.

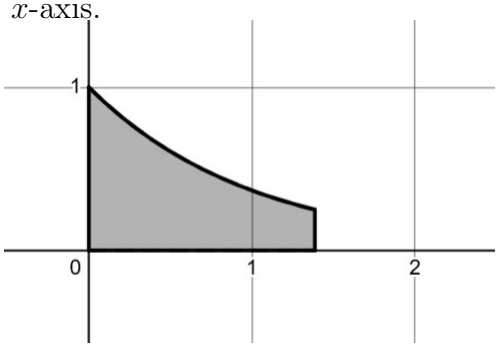
Theorem. (Washer Method) Let f and g be continuous functions with $f(x) \geq g(x) \geq 0$ on $[a, b]$. Let R be the region bounded by the graphs $y = g(x)$ and $y = g(x)$ from a to b. When R is revolved about the x-axis, the resulting solid's cross section at $x = c$ is a washer with area

$$A(c) = \pi(f(c)^2 - g(c)^2)$$

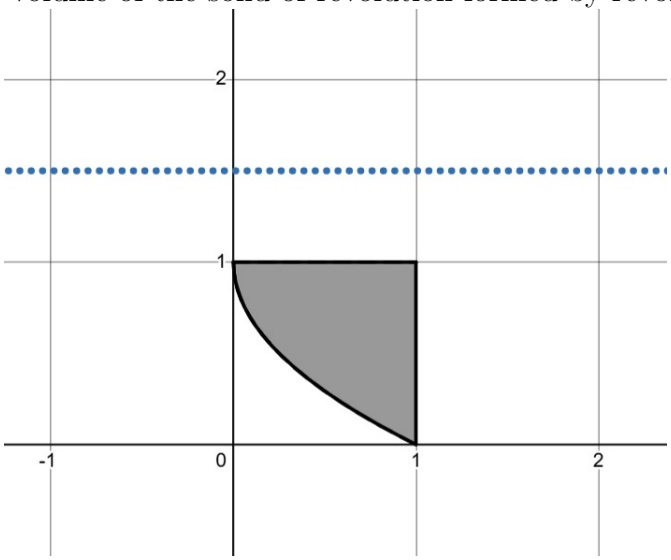
and the volume of the solid of revolution is

$$V = \int_a^b A(x)dx = \int_a^b \pi(f(x)^2 - g(x)^2)dx.$$

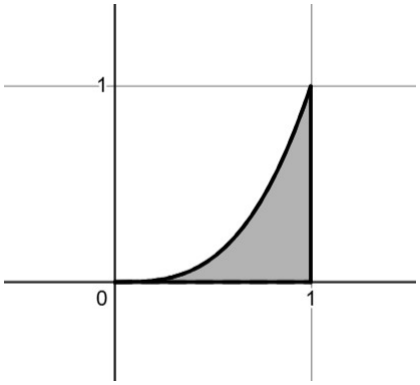
Example 6.3.3. Let R be the region bounded by the curves $y = e^{-x}$, $y = 0$, $x = 0$, and $x = \ln 4$. Find the volume of the solid of revolution formed by revolving R about the x-axis.



Example 6.3.4. Let R be the region bounded by $y = 1 - \sqrt{x}$, $x = 1$, and $y = 1$. Find the volume of the solid of revolution formed by revolving R about the line $y = 1.5$.



Example 6.3.5. Let R be the region bounded by the curves $y = x^3$, $y = 0$, and $x = 1$. Find the volume of the solid of revolution formed by revolving R about the y-axis.



Example 6.3.6. Let R be the region bounded by the curves $y = 2 - x$, $y = 1 - x$, $x = 0$, and $y = 0$. Find the volume of the solid of revolution formed by revolving R about the x-axis.

